

## MODULE SPECIFICATION

<b>Module Title</b>	Mathematical Statistics and Probability
<b>Module Code</b>	STAT 472
<b>Credits</b>	4
<b>Pre-requisites</b> ( <i>including Year 1</i> )	Calculus III and Introduction to Probability and Statistics

### Description

<b>Course Overview</b>
<p>This course provides a rigorous study of mathematical statistics and statistical inference built upon probability theory. Topics include multivariate probability distributions, transformations of random variables, moment generating functions, order statistics, estimation theory, sufficiency, confidence intervals, hypothesis testing, likelihood methods, asymptotic theory, and optimal testing procedures.</p> <p>The course emphasizes mathematical derivations, statistical reasoning, and theoretical foundations necessary for advanced work in statistics, machine learning, econometrics, engineering, actuarial science, quantitative finance, and data science.</p> <p>The course is intended for students prepared for proof-oriented reasoning and calculus-based statistical analysis.</p> <p><b>Course Learning Outcomes</b></p> <p>By the end of the course, students will be able to:</p> <ol style="list-style-type: none"> <li>1. Analyze discrete and continuous probability models using formal mathematical methods.</li> <li>2. Work with joint, marginal, conditional, and multivariate probability distributions.</li> <li>3. Apply transformations of random variables in both univariate and multivariate settings.</li> <li>4. Use moment generating functions and characteristic functions to derive distributions and moments.</li> <li>5. Apply major probability distributions including gamma, beta, chi-square, multivariate normal, t, and F distributions.</li> <li>6. Derive and evaluate estimators using Method of Moments and Maximum Likelihood methods.</li> <li>7. Analyze properties of estimators including unbiasedness, consistency, efficiency, and sufficiency.</li> <li>8. Construct confidence intervals and perform classical hypothesis testing procedures.</li> <li>9. Apply likelihood ratio methods and optimal testing procedures.</li> </ol>

10. Use asymptotic theory including LLN, CLT, Slutsky's theorem, and Delta Method.
11. Communicate statistical reasoning clearly using rigorous mathematical notation and derivations.

## **Method of Teaching and Learning**

This module will be taught using a combination of lectures, tutorials and consultation hours. Learning will also be reinforced by appropriate readings from the course text.

## **Syllabus**

Modules

### **Module 1 – Foundations of Mathematical Statistics**

#### **Topics**

- Introduction to mathematical statistics
- Sets and algebra of events
- Probability axioms
- Conditional probability
- Independence
- Random variables
- Discrete random variables
- Continuous random variables
- Distribution functions
- Density and mass functions
- Expectation and moments
- Variance and covariance
- Special expectations
- Important inequalities
- Moment generating functions
- Characteristic functions

#### **Textbook Coverage**

- Chapters 1-3

### **Module 2 – Joint and Multivariate Distributions**

#### **Topics**

- Joint probability distributions
- Marginal distributions
- Conditional distributions
- Conditional expectation
- Joint density functions
- Independence of random variables
- Covariance and correlation
- Correlation coefficient
- Linear combinations of random variables
- Conditional variance
- Extension to several random variables
- Multivariate distributions
- Covariance matrices

#### **Textbook Coverage**

- Chapters 3-4

### **Module 3 – Functions and Transformations of Random Variables**

**Topics**

- Functions of a single random variable
- Jacobian transformations
- Transformations for bivariate random variables
- Transformations for multivariate random variables
- Sums of independent random variables
- Convolutions
- Distribution of functions of random variables
- Order statistics
- Sample minima and maxima
- Sample quantiles
- Linear transformations
- Quadratic forms

**Textbook Coverage**

- Chapters 4-5

**Module 4 – Important Probability Distributions****Topics**

- Binomial distribution
- Poisson distribution
- Hypergeometric distribution
- Negative binomial distribution
- Gamma distribution
- Exponential distribution
- Beta distribution
- Chi-square distribution
- Normal distribution
- Multivariate normal distribution
- t-distribution
- F-distribution
- Mixture distributions
- Exponential family distributions

**Textbook Coverage**

- Chapters 5-6

**Module 5 – Sampling Distributions and Asymptotic Theory****Topics**

- Random sampling
- Sampling distributions
- Statistics and estimators
- Law of Large Numbers
- Convergence in probability
- Convergence in distribution
- Central Limit Theorem
- Multivariate CLT extensions
- Continuous Mapping Theorem
- Slutsky's Theorem
- Delta Method
- Monte Carlo methods
- Bootstrap procedures

- Tolerance limits

**Textbook Coverage**

- Chapters 7-8

**Module 6 – Point Estimation**

**Topics**

- Point estimation
- Properties of estimators
- Unbiasedness
- Consistency
- Efficiency
- Mean squared error
- Relative efficiency
- Method of Moments estimation
- Maximum Likelihood Estimation
- Likelihood equations
- Multiparameter estimation
- Invariance property of MLEs
- Fisher Information
- Rao-Cramér lower bound
- Bayesian estimation
- EM Algorithm (introduction and applications)

**Textbook Coverage**

- Chapters 8-10

**Module 7 – Sufficiency and Optimal Estimation**

**Topics**

- Sufficient statistics
- Factorization theorem
- Minimal sufficiency
- Complete statistics
- Completeness and uniqueness
- Rao-Blackwell theorem
- Lehmann-Scheffé theorem
- UMVU estimators
- Ancillary statistics
- Sufficiency and independence
- Functions of parameters
- Several-parameter cases
- Exponential class distributions

**Textbook Coverage**

- Chapters 10-11

**Module 8 – Interval Estimation**

**Topics**

- Confidence intervals
- Pivotal quantities
- Confidence intervals for means
- Confidence intervals for variances
- Confidence intervals for proportions

- Confidence intervals for discrete distributions
- Likelihood-based confidence intervals
- Large-sample confidence intervals
- Simultaneous confidence intervals

**Textbook Coverage**

- Chapters 8-9

**Module 9 – Hypothesis Testing and Optimal Tests**

**Topics**

- Statistical hypotheses
- Type I and Type II errors
- Significance levels
- p-values
- Power functions
- Neyman-Pearson Lemma
- Most powerful tests
- Uniformly most powerful tests
- Likelihood ratio tests
- Likelihood ratio tests for variances
- Multiparameter hypothesis testing
- Bayesian testing
- Chi-square tests
- Sequential probability ratio tests
- Minimax procedures
- Classification procedures
- Composite hypothesis testing

**Textbook Coverage**

- Chapters 12-14

**Assessment**

Assessment Type	% of Final Mark
Midterm 1	25%
Midterm 2	25%
Final Exam	30%
Homework & Quizzes	10%
Course Participation	10%

<i>Range</i>	<i>Letter Grade</i>
90% - 100%	A
80% - 89%	B
70% - 79%	C
60% - 69%	D
< 60%	U

## **Textbooks**

### *Mandatory Textbooks*

<b>Title</b>	<b>Editor/Author</b>	<b>ISBN/Publisher</b>
<i>Introduction to Mathematical Statistics</i> (8th ed.).	Hogg, R. V., McKean, J., & Craig, A. T.	Pearson

### *Optional Textbooks*

<b>Title</b>	<b>Author</b>	<b>ISBN/Publisher</b>
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### *Reference Textbooks*

<b>Title</b>	<b>Author</b>	<b>ISBN/Publisher</b>
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